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Prediction of interior noise in buildings generated by underground rail traffic

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Abstract

The prediction of sound field in cavities surrounded by vibrating walls is a simple task nowadays, provided that the velocity distribution along the walls is known in sufficient detail. This information can be obtained from a structural finite element (FE) calculation of the building and the results can be fed directly into a conventional boundary element (BE) analysis. Though methodically simple, it is not an attractive way of prediction from the practical point of view: the size of the matrices needed for BE calculation is too large, thus their inversion is very cumbersome and computationally intensive.

The paper introduces a modified numerical calculation method appropriate for practical calculations without the need to construct and invert large matrices. The suggested method is based on the Rayleigh radiation integral and some standard direct (collocational) BE techniques, where the necessary input data are generated from measured or calculated velocity values at just a few points. The technique has been compared and validated on the basis of an extensive measurement series, performed in a reinforced concrete frame building close to a tunnel of line RER B of the underground railway network in Paris.

1. Introduction

It is a typical task of the acoustic engineer to predict interior noise levels generated by structural vibrations. The source of structural vibration may be either a railway line running near a building or a machine inside the building which is not properly vibration isolated. For the prediction of reradiated noise detailed information on the vibration distribution along the boundary surfaces is needed. This can be determined by using finite element modelling (FEM). The prediction of the vibration levels in buildings is not discussed in this paper.

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To predict the noise from the vibration velocities, the most accurate way is to use the boundary element method (BEM) [1–3]. The advantage of the BEM is that it can be applied to arbitrary shaped enclosures and it can easily be connected to the structural FE model of the building. The disadvantage is that the BEM is a low-frequency method. In the case of a typical room, for calculations in the mid-frequency range, the density of the boundary mesh should be increased and that would result in a large system of linear equations, which could be very demanding from the computation point of view.

Another possibility for predicting reradiated noise is to use empirical formulae (such as the one proposed by Kurzweil [4]). The advantage of these formulae is their simplicity, since in practice predictions of environmental impact are based on more generalized vibration data, rather than on detailed vibration distributions. Of course, the main disadvantage of these methods is a consequence of their approach: they give only very approximate results.

Statistical energy analysis (SEA) is also a popular method for calculating structure-borne noise in buildings. It is also based on much coarser vibration data of the walls, i.e. the mean squared vibration level. The simplified analytical formula originating from SEA uses the radiation efficiency of walls as an input. Unfortunately, radiation efficiency varies rapidly in the low-frequency range and is hard to determine.

To solve exterior radiation problems, Rayleigh integral-based methods are well known and widely used. In this paper, a new and simple way to predict interior noise levels is presented. The new method is based on the Rayleigh integral that is combined with room acoustics modelling. It uses the vibration and geometrical data of the room, so it can be easily coupled to a FE model, and there is no need of knowledge of the radiation efficiency. Compared with the BEM, the matrix describing the sound field is smaller and it is not necessary to invert it.

2. Description of Rayleigh integral-based method

If the surface-normal component $v_n(\mathbf{r},\omega)$ of the vibration velocity distribution on an infinite plane is known, the Rayleigh integral can be used to calculate the radiated sound pressure $p(\mathbf{q},\omega)$ at an arbitrary point $\mathbf{q}$ of the space [5,6]:

$$p(\mathbf{q},\omega) = \iint_{\Sigma} g(\mathbf{r},\mathbf{q},\omega) v_n(\mathbf{r},\omega)\,\mathrm{d}\Sigma, \tag{1}$$

where

$$g(\mathbf{r},\mathbf{q},\omega) = \frac{\mathrm{e}^{-\mathrm{j}k|\mathbf{q}-\mathbf{r}|}}{|\mathbf{q}-\mathbf{r}|} \tag{2}$$

is the Green's function of the homogeneous acoustical full-space, $k=\omega/c$ is the wavenumber, ω is the circular frequency, and c is the speed of sound.

If the Rayleigh integral is applied to the case of closed spaces, an appropriate estimate of the radiated sound pressure is obtained. The result is the sum of sound pressures radiated by each wall into an acoustic half-space. Each wall is handled separately as part of an infinite plane. Only a small part of the infinite wall vibrates, the surface-normal vibration velocity $v_n(\mathbf{r},\omega)$ along this part is known. Outside the vibrating part $v_n=0$ is taken:

$$p(\mathbf{q},\omega) = \sum_{n=1}^{N} \iint_{\Sigma_n} g(\mathbf{r},\mathbf{q},\omega) v_n(\mathbf{r},\omega)\,\mathrm{d}\Sigma, \tag{3}$$

where N is the number of walls.

To take into account not only the primarily radiated sound but the reflections from the walls, the Green's function must be modified (2). Each reflection can be handled as an image source, the location of which is determined by mirroring the originally radiating part of wall in the reflecting walls, and its amplitude is reduced proportionally to the absorption at the reflections:

$$\hat{g}(\mathbf{r}_0, \mathbf{q}, \omega) = \frac{e^{-jk|\mathbf{q}-\mathbf{r}_0|}}{|\mathbf{q}-\mathbf{r}_0|} + \gamma_1(\omega)\frac{e^{-jk|\mathbf{q}-\mathbf{r}_1|}}{|\mathbf{q}-\mathbf{r}_1|} + \cdots + \gamma_i(\omega)\frac{e^{-jk|\mathbf{q}-\mathbf{r}_i|}}{|\mathbf{q}-\mathbf{r}_i|} + \cdots, \quad (4)$$

where $\mathbf{r}_i$ is the location of the ith image source and $\gamma_i(\omega)$ represents the effect of absorption.

This method is rather complicated in the general case, since a large number of image sources has to be taken into account, and their location and visibility (validity) have to be determined individually.

For simplicity, the investigations here are restricted to the case of cuboid rooms. This restriction allows visibility tests to be neglected and the determination of the location of the image sources can be easily automated.

In the following it is assumed that the absorption of the walls is constant $\Gamma(\omega)$. With this Eq. (4) can be converted to the following form:

$$\hat{g}(\mathbf{r}_0, \mathbf{q}, \omega) = \frac{e^{-jk|\mathbf{q}-\mathbf{r}_0|}}{|\mathbf{q}-\mathbf{r}_0|} + \Gamma(\omega)\sum_{k=1}^{K_1}\frac{e^{-jk|\mathbf{q}-\mathbf{r}_k|}}{|\mathbf{q}-\mathbf{r}_k|} + \cdots + \Gamma(\omega)^i\sum_{k=1}^{K_i}\frac{e^{-jk|\mathbf{q}-\mathbf{r}_k|}}{|\mathbf{q}-\mathbf{r}_k|} + \cdots, \quad (5)$$

where K_i is the number of the ith order reflections. In practice, the order of reflections is limited and Eq. (5) can be expressed as a finite summation.

The surface integrals in Eq. (3) are performed numerically. The whole surface is divided into surface elements of constant velocity distribution. The integration of the Green's function over the elements can be carried out by means of Gaussian quadrature integration. The discretized form of Eq. (3) is

$$p(\mathbf{q}, \omega) = \sum_{i=1}^{N_e} v_{n_i}(\omega)\sum_{s=1}^{N_p} w_s\hat{g}(\mathbf{r}_{is}, \mathbf{q}, \omega), \quad (6)$$

where N_e is the number of elements, N_p is the number of Gaussian points, v_{n_i} is the surface-normal velocity of the ith element, w_s is the Gaussian weight and r_{is} is the location of the sth Gaussian point on the ith element.

3. Numerical testing results

To verify and validate the newly designed prediction method, numerical tests were performed by means of a cuboid room of size 4 m × 5 m × 3 m. The method of numerical testing was to calculate the internal noise at different points by means of the direct BEM and the Rayleigh-based method, assuming a normal velocity distribution on the walls [7].

In the first series of tests a simple case was examined to investigate the convergence of the new method. One element of the mesh was chosen as the only vibrating wall section of the test-room and the sound field generated by this single element was calculated for different absorption coefficients and taking different numbers of reflections into account. Absorption was changed from 0% to 50% and the number of reflections was increased from 0 to 9261 (zero reflections means applying the original Rayleigh integral to the walls). The results showed what had been expected: with increasing absorption the fluctuations in the response decrease, and by increasing the number of reflections, the modal behaviour of the room can be observed more and more clearly.

In the second series of tests the effects of spectral and spatial averaging ('dithering') were examined for several velocity distributions along the walls. Figs. 1 and 2 show narrow and one-third octave band spectra of noise levels calculated at seven points located closely around one internal point, assuming constant velocity distribution all over the room's surface. The maximum distance between two points is 0.2 m. The solid line shows the one-third octave band spectrum averaged over the seven points. It can be clearly seen that although the narrow band spectra can show significantly different levels (mostly at the peaks and dips), the smooth curves obtained by spectral averaging are similar for both methods. The necessity of spatial averaging is also shown.

Figs. 3 and 4 display narrow and one-third octave band spectra at three points (averaged from 3 × 7 points) inside the enclosure assuming a more complex velocity distribution. The modal behaviour of the room is

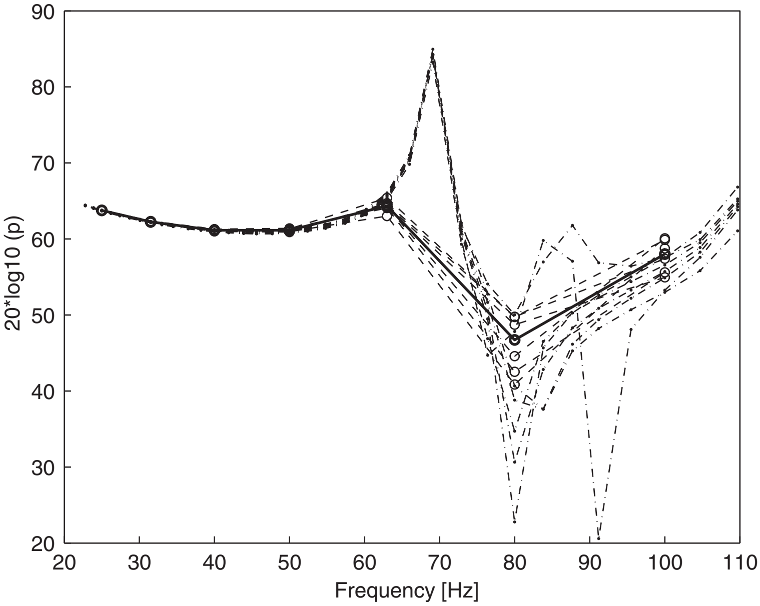

Fig. 1. The effect of spectral and spatial averaging on boundary element method. -.-.-. Narrow band spectra at 7 points located closely around the internal point, - - - one-third octave band spectra at the same 7 points, —— one-third octave band spatially averaged spectrum.

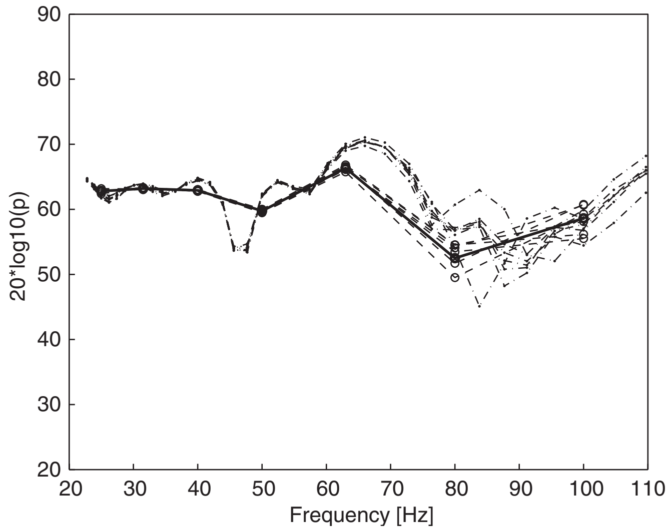

Fig. 2. The effect of spectral and spatial averaging on the Rayleigh-based method. -.-.-. Narrow band spectra at 7 points located closely around the internal point, - - - one-third octave band spectra at the same 7 points, —— one-third octave band spectrum averaged spatially.

clearly shown by the narrow band spectra calculated by the two methods, and the spectral averaging allows similar results to be obtained not only in terms of the modal frequencies, but also, in noise levels.

4. Experimental validation

4.1. Noise and vibration measurements

For the experimental validation of the newly developed technique, an extensive measurement programme was performed in a concrete building close to the tunnel of line RER B of the underground railway line network in Paris.

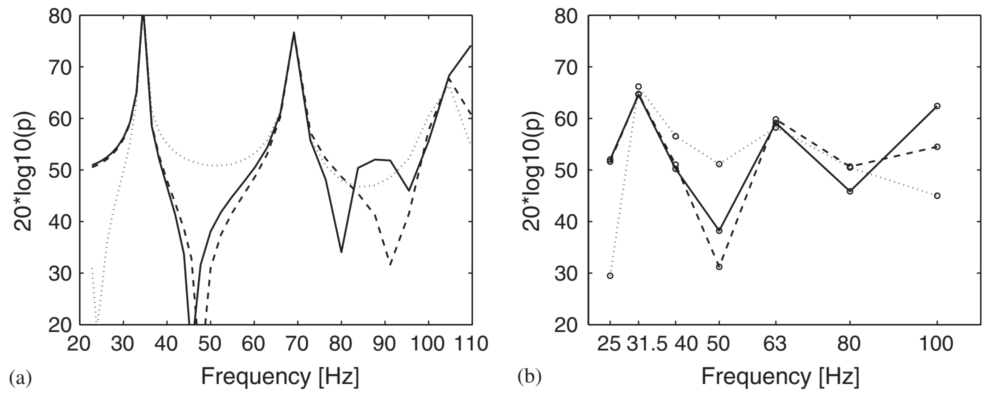

Fig. 3. Results of boundary element calculation at different internal points assuming modal normal velocity distribution. —— Point 1, - - - point 2, ... point 3. (a) Narrow band, (b) one-third octave band spectra.

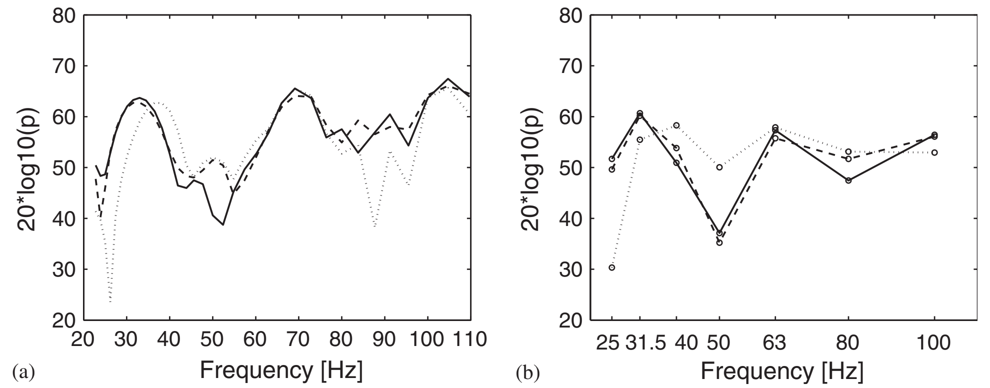

Fig. 4. Results of the Rayleigh-based calculation at different internal points assuming modal normal velocity distribution. —— Point 1, - - - point 2, ... point 3. (a) Narrow band, (b) one-third octave band spectra.

A room of dimensions $4.5\,m \times 5.8\,m \times 3.4\,m$ was selected in the basement of the building with little furniture and clear concrete walls (Fig. 5). By using a parallel measurement system of 16 vibration and 6 noise channels, more than 80 train passbys were recorded, out of which 62 were good enough to be evaluated. Sound pressures were measured at 6 points, vibration was measured altogether at 56 points plus one reference point, providing a good quality data set suitable for model validations. Vibration and noise levels during a typical train passby can be seen in Fig. 6.

4.2. Numerical modelling

As input for the BEM and Rayleigh-based calculations, a dense boundary mesh is required with velocity values at each node of the mesh. Linear interpolation was used to get the vibration distribution along the walls from the 57 measurement points. The mesh for interpolation can be seen in Fig. 7, and the resulting boundary mesh representing the vibration values is shown in Fig. 8. The results of experimental modal analysis were also used for the determination of the vibration distribution of the ceiling.

4.3. Predictions by means of empirical formula and SEA

For comparison, simple calculations were also performed on the same room using the same vibration data. First, the simple formula of Kurzweil [4] was applied:

$$L_p = L_a - 20\log_{10} f + 37, \tag{7}$$

Fig. 5. The measurement site in the basement with accelerometers on the ceiling.

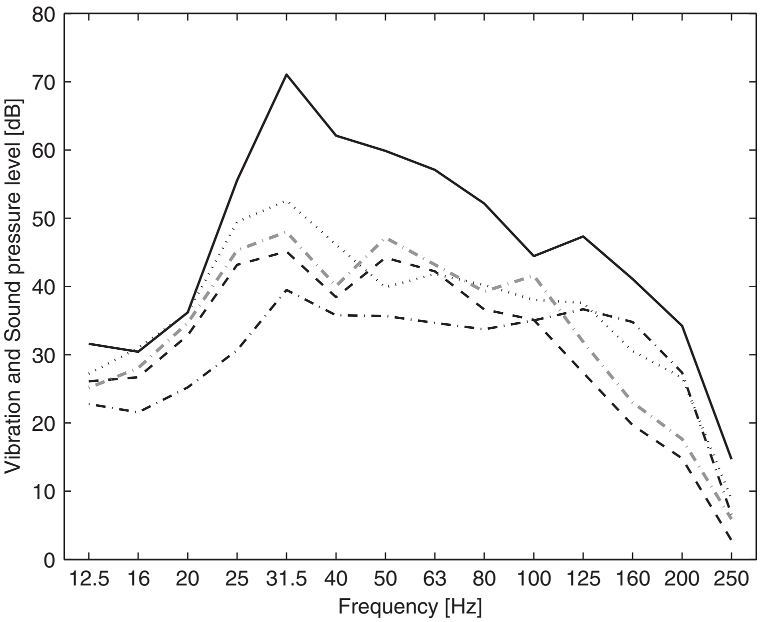

Fig. 6. Typical one-third octave band vibration and sound pressure levels during train passby. —— Sound pressure level (Ref 2×10^{-5} Pa), - - -, ... and -.-.-. vibration levels (Ref 5×10^{-8} m/s).

where L_a is the octave band room floor acceleration level, L_p is the resulting sound octave band pressure level in the room and f is the octave band centre frequency. The formula, although originally given for octave bands, was used to calculate the sound pressure level in one-third octave bands. First, the average floor acceleration was taken as vibration input L_a in order to comply with Ref. [4].

According to our measurements, the surface with the highest vibration level was not the floor. Therefore, for the second calculation the vibration level averaged over the room was taken as input.

For SEA calculations, the FreeSEA [8] software was used. The model of the room was very simple: one cavity and six walls that are connected only to the cavity and not to each other. The average wall vibration

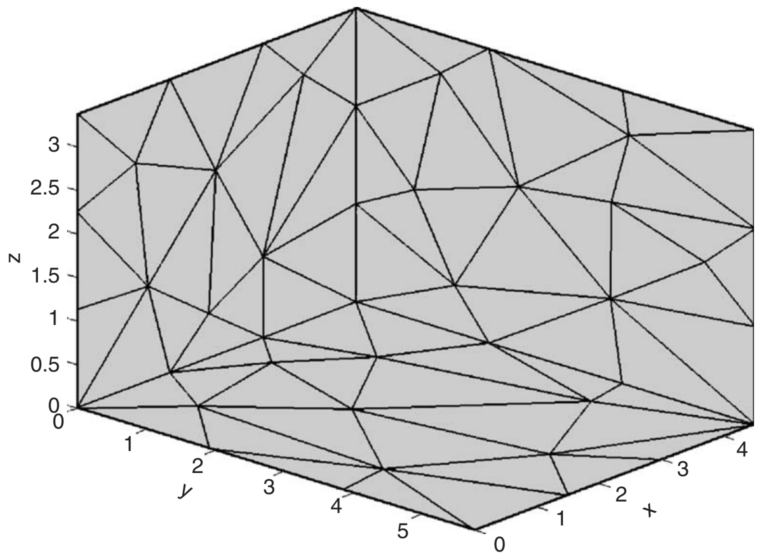

Fig. 7. The interpolation mesh used to generate vibration distribution input for the calculations.

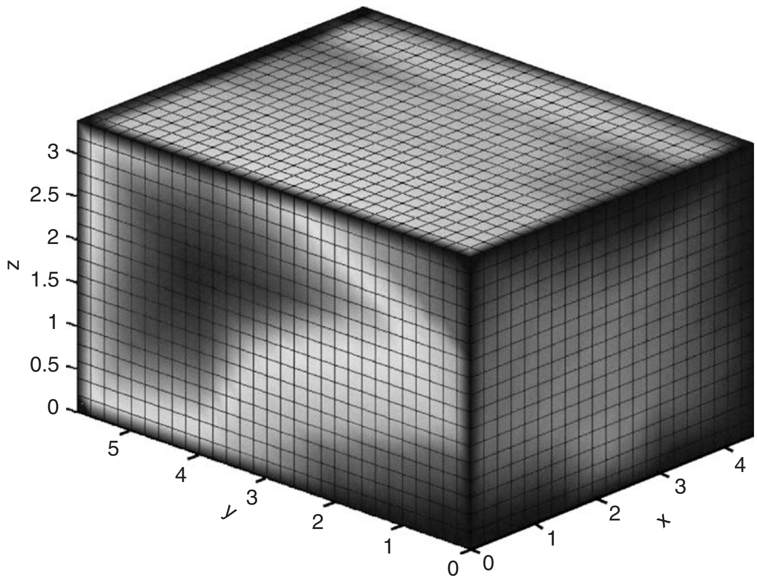

Fig. 8. Vibration distribution along the boundary element mesh used as input for the calculations.

levels were calculated from the measurements, whilst the reverberation time of the room was set to be constant with two different values (1 and 2.5 s).

The formula originating from SEA theory gives radiated sound power:

$$P = \sum_{i}^{N_w} \rho_0 c A_i v_i^2 \sigma_i, \tag{8}$$

where N_w is the number of walls, $\rho_0 c$ is the characteristic impedance of air, A_i is the surface area, v_i is the average vibration velocity and σ_i is the radiation efficiency of the ith wall. For the calculation the same inputs

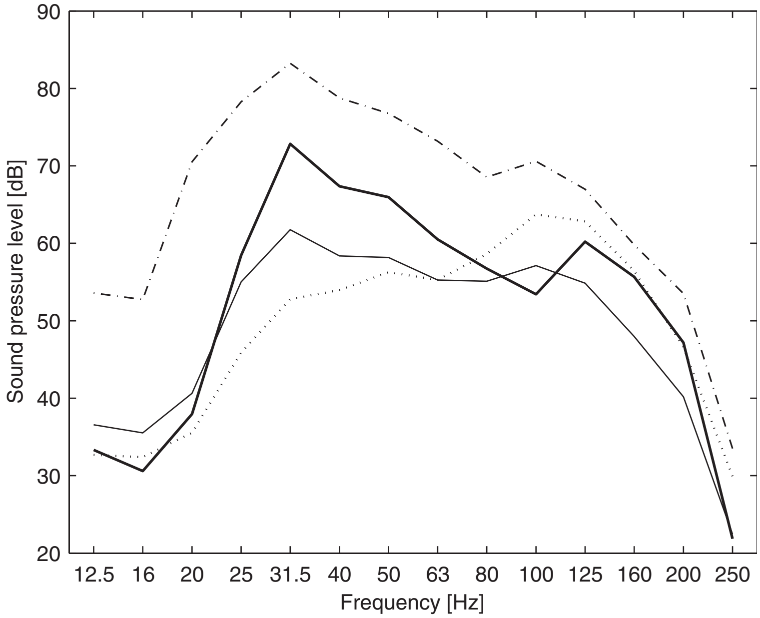

Fig. 9. Results of calculations using Kurzweil's formula. Solid thick line shows measured results, thin lines show results of calculations. —— Using the room average vibration levels as input, -.-.-. using the maximum vibration levels as input, ... using the floor vibrations as input.

as for SEA were used and $\sigma = 1$ was taken at all frequencies. The calculated sound power was transformed into sound pressure assuming several different average absorption coefficients (to achieve the same reverberation time as for the SEA modelling). To compare with the Kurzweil result a second calculation was performed with vibrations along the floor only.

The results of the calculations are shown in Fig. 9 (Kurzweil), Fig. 10 (SEA) and Fig. 11 (analytical), together with the measured values.

Using the formula of Kurzweil the calculated sound levels were lower than the measured values when taking the average vibration levels as input. When taking the maximum levels, the result was much higher than the measured values. Unfortunately, handling more than one radiating surface is not discussed in Ref. [4].

In the SEA calculations, all radiating surfaces could be taken into account. In the higher frequency range the calculations overestimated the measured values, although the assumed wall absorption was higher (and thus the reverberation time was lower) than in reality.

As expected, the results of the analytical formula are similar to that of SEA. The largest difference occurred at low frequencies, which is likely to be due to the radiation efficiency, that was set to 1 in the analytical calculations.

It is worth noting that, when calculating the noise only from floor vibrations, the results of the simple Kurzweil formula and of the more sophisticated analytical formula do not differ too much.

In practice, when there is only little data available on the vibration of walls, these simple calculations can be used to predict radiated noise. As can be seen, these formulae give only a rough and inappropriate estimate of the noise level.

4.4. Comparison of numerical modelling results

Fig. 12 shows the results of boundary element modelling at one of the six microphone positions compared to the measured values, whilst Fig. 13 shows the results of the Rayleigh-based calculation (with different numbers of reflection order).

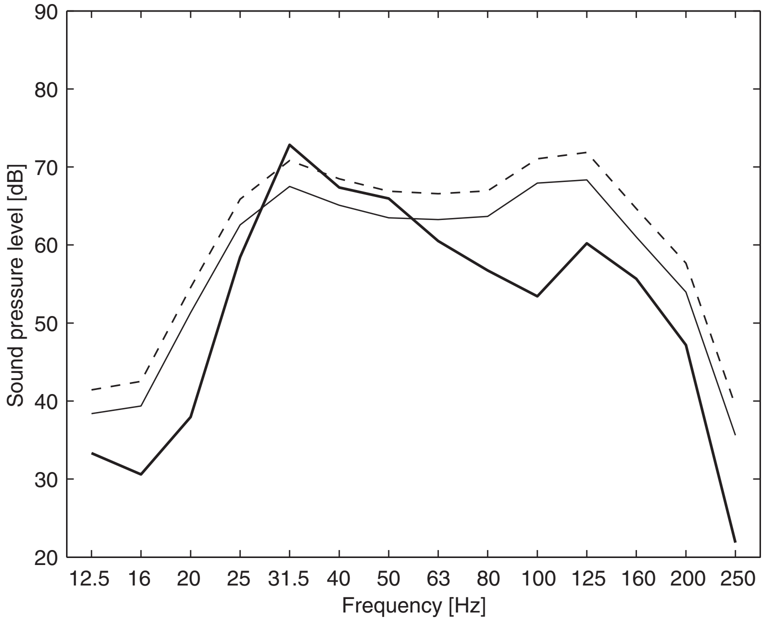

Fig. 10. Results of calculations using statistical energy analysis. Solid thick line shows measured results, thin lines show results of calculations. —— Reverberation time = 1 s, - - - reverberation time = 2.5 s.

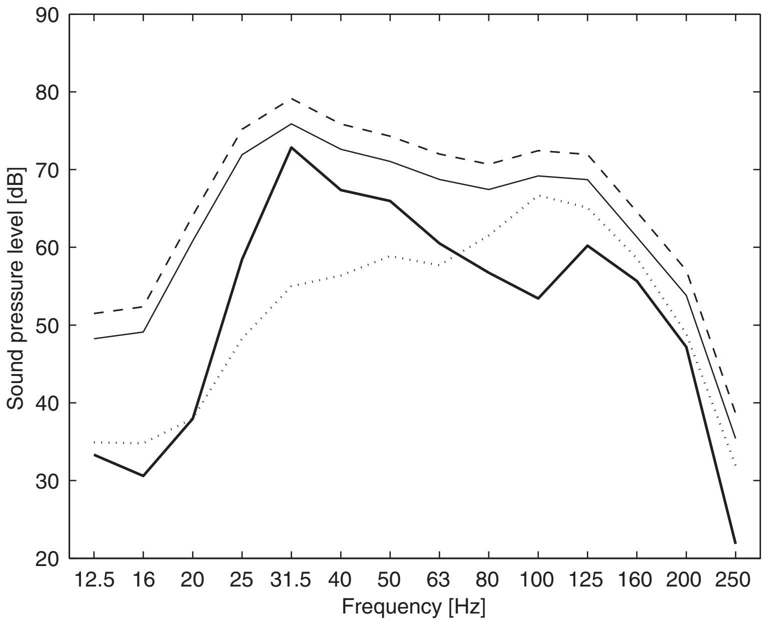

Fig. 11. Results of calculations using the analytical formula. Solid thick line shows measured results, thin lines show results of calculations. —— Reverberation time = 1 s, - - - reverberation time = 2.5 s, ... using only the floor vibrations as input.

It can be seen that both methods give a reasonably good approximation to the measured sound level, but differences between the results can reach up to 8–10 dB at certain frequencies. If the results are expressed in equivalent A-weighted levels, the Rayleigh-based method's lowest error was a difference of 1.3 dB and the highest error 3.4 dB, whilst that of the BEM was 0.4 dB and 1.2 dB, respectively.

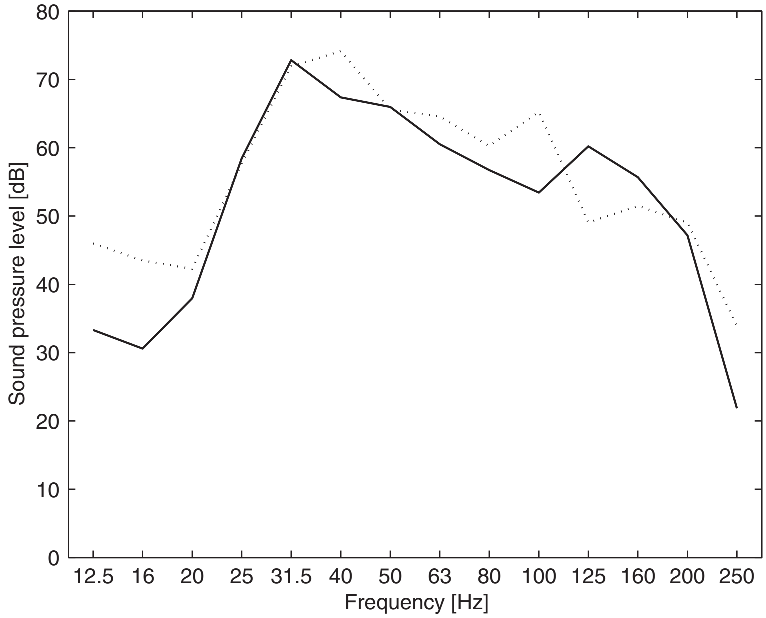

Fig. 12. Results of the standard collocational boundary element analysis compared to the measurement results. —— Measured, ... calculated.

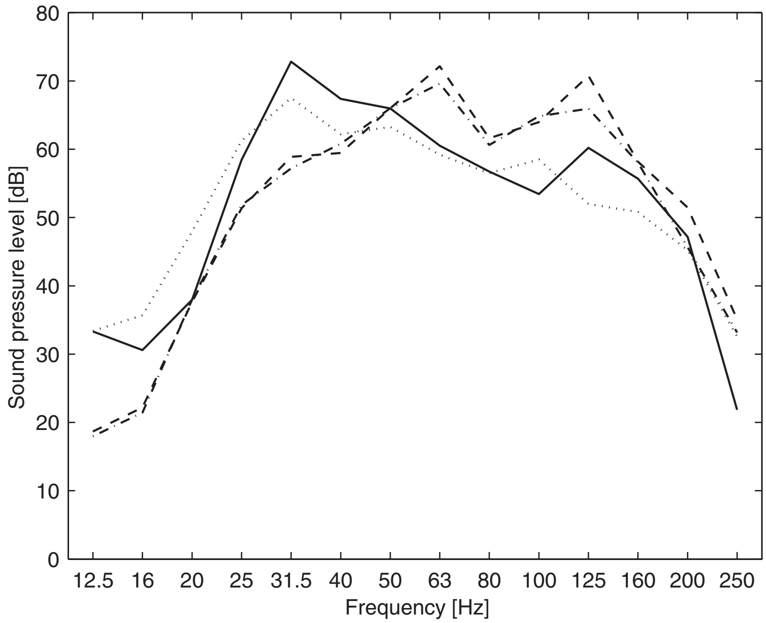

Fig. 13. Results of the Rayleigh-based calculation compared to the measurement results. —— Measured, - - -, ... and -.-.-. Rayleigh-based calculation with different number of reflections taken into account.

5. Conclusion

In this paper a newly devised, simple method for predicting radiated noise levels in enclosures has been presented. The method is based on the Rayleigh integral and uses a modified Green's function, which gives the sound field at an arbitrary point in a cuboid room, due to a point source placed on the boundary surface. It is also assumed that acoustic absorption is evenly distributed over the walls of the room and can be described by a constant, frequency-dependent absorption coefficient.

The method was verified by comparison with the conventional collocational BEM, on the basis of both numerical experiments and an extensive programme of parallel noise and vibration measurements under realistic conditions. Investigations have shown that the results of the two methods are in good agreement, provided that both spatial and spectral averaging is used. Compared to BEM, the proposed Rayleigh-based method gives reliable results without the disadvantage of creating and inverting large matrices.

Compared to more generalized calculations such as SEA or the formula of Kurzweil, it can be stated that by using numerical methods such as the BEM or the proposed Rayleigh-based method, the prediction of the noise from vibration could be accurate if the vibration prediction were accurate and sufficiently detailed.

As the BEM, the Rayleigh-based method can be used not only for rooms but for any other cavities to calculate the sound radiated from its walls. The method in its present state can be applied only to cuboid enclosures. The main aim of future research is to develop a method that handles arbitrary shaped cavities and thus to broaden the field of its applicability. The other goal is to introduce some statistical considerations in order to achieve a procedure for predicting noise in enclosures where vibration data is only available in a small number of points. With these improvements, the Rayleigh-based method would become a real alternative of the BEM.

Acknowledgements

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